**12 Physics Forces, energy and motion modelling task**

### Unit 3 Outcome 1

### Investigate motion and related energy transformations experimentally, and analyse motion using Newton’s laws of motion in one and two dimensions

### Task Outline

In this task you will apply your understanding of the physics concepts of motion, forces and energy to explain how the device, a catapult, can be used to propel a cart by the transformation of elastic potential energy.

### Relevant Key knowledge descriptors

* analyse transformations of energy between kinetic energy, elastic potential energy, gravitational potential energy and energy dissipated to the environment (considered as a combination of heat, sound and deformation of material):
* kinetic energy at low speeds: $E\_{k}=\frac{1}{2}mv^{2}$; elastic and inelastic collisions with reference to conservation of kinetic energy
* elastic potential energy: area under force-distance graph including ideal springs obeying Hooke’s Law: $E\_{s}=\frac{1}{2}kx^{2}$
* gravitational potential energy: *E*g = *mg*Δ*h* or from area under a force-distance graph and area under a field-distance graph multiplied by mass.

### Marking Criteria

 28 marks allocated

### Introduction

### A student designed a catapult to demonstrate how elastic potential energy can be used to propel a cart forward. The student modelled the cart as a block with a mass of 0.28 kg which was placed inside a rubber band to two nails. To propel the cart forward, it was pulled it back horizontally a given distance and then released. They recalled that elastic potential energy can be determined using the formula, *Es* = $\frac{1}{2}kx^{2}$, and the force that the rubber band produces can be calculated using the formula, *Fs = - k*$∆x$*.* Often in calculations the negative value is not included as you can calculate the magnitude and assign the sign based on the direction of the force. The student found that the spring constant, *k*, is a measure of the strength of the elastic band, while *x* is the distance the rubber band is stretched in metres. They were able to select from a range of rubber bands with *k* values given in Table 1 and they stretched the rubber band 0.22 m.

Table 1.

|  |  |
| --- | --- |
| rubber band | *k* (N.m-1) |
| 1 | 15.8 |
| 2 | 30.7 |
| 3 | 81.3 |
| 4 | 100 |

m = 0.28 kg

*x*

m = 0.28 kg

*x*

## 1. Conservation of energy

The energy required to propel the cart forward is an outcome of the transformation of elastic potential energy to kinetic energy.

1. Explain how this energy transformation results in the forward motion of the cart. Assume the system is frictionless

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
| **demonstrates understanding** | identifies concepts and/or formulae | explains concepts and/or formulae | applies concepts and/or formula to context  | connects concepts and/or formula |

- identifies EPE 🡪 KE

- explains how elastic potential energy is stored in springs or rubber bands by stretching or compressing the spring

- explains how kinetic energy is connected to movement e.g. mentioned velocity and/or shows the equations

- connects the concepts to the context by relating how elastic potential energy is stored in the catapult by stretching the rubber band backwards and then on release this is transformed to kinetic energy which provides movement.

- connects the concepts together by demonstrating how elastic potential energy is converted to kinetic energy which then causes movement.

1. Assign numerical values and use relevant mathematical relationships to model how the change in energy identified in part a) results in the forward motion of the car. Select one of the rubber bands given in table 1 for your model and calculate the velocity of the cart as it is released from the catapult.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
| **mathematically models energy transformation** | identifies mathematical relationships | inserts appropriate numerical values | calculates the velocity of the car | evaluates the limitations of the model |

*Es* = $\frac{1}{2}kx^{2}$

rubber band 1, k = 15.8 N.m-1 Es1 = 0.5 x 15.8 x 0.222 = 0.382 J

rubber band 2, k = 30.7 N.m-1 Es2 = 0.5 x 30.7 x 0.222 = 0.743 J

rubber band 3, k = 81.3 N.m-1 Es3 = 0.5 x 81.3 x 0.222 = 1.967 J

rubber band 4, k = 100 N.m-1 Es4 = 0.5 x 100 x 0.222 = 2.42 J

Es = Ek,

rubber band 1, Es1 = Ek1= 0.382 J, v2 = 2x 0.382/0.28, v = 1.65 m.s-1

rubber band 2, Es2 = Ek1= 0.743 J, v2 = 2x 0.743/0.28, v = 2.29 m.s-1

rubber band 3,Es3 = Ek1= 1.967 J, v2 = 2x 1.967/0.28, v = 3.75 m.s-1

rubber band 4, Es4 = Ek1= 2.42 J, v2 = 2x 2.42/0.28, v = 4.16 m.s-1

Limitations of the model

In a real context the system would not be frictionless.

Therefore, Es $\ne $ Ek as some energy is lost as thermal energy, sound energy etc.

Therefore, Ek would be lower, and the velocity will be correspondingly lower than the values calculated.

1. What changes could you make to the design of the rubber band car to increase the cart’s velocity? Justify your modifications by assigning numerical values and using relevant mathematical relationships to model the effect of the changes.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
| **mathematically models modifications** | identifies mathematical relationships | inserts appropriate numerical values | compares modifications with original value | evaluates which modification is most effective |

Students could change the;

* *k* value
* *x* value
* mass of the block
* add another rubber band

They need to repeat their calculations

Then state compare values and make a statement about which modification is most effective.

# 2. Forces and inclined planes

1. The student then placed the catapult on an incline of 23o and again pulled the cart back by 0.22 m. Calculate the applied force on the cart when the rubber band is released and the car starts moving down the ramp. Indicate which rubber band you selected to use for your calculations. Assume the system has no friction and all of the gravitational potential energy and elastic potential energy is transformed to kinetic energy. Let g = 10 m.s-2.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
| **mathematically models net force** | identifies mathematical relationships | inserts appropriate numerical values | calculates the applied force on the car | evaluates the limitations of the model |

 Applied force on cart = FII + Fs

0.45 m m

23o

Calculate Fs = kx,

rubber band 1, k = 15.8 N.m-1 Fs = kx = 15.8 x 0.22 = 3.476 N

rubber band 2, k = 30.7 N.m-1 Fs = kx = 30.7 x 0.22 = 6.754 N

rubber band 3, k = 81.3 N.m-1 Fs = kx = 81.3 x 0.22 = 17.89 N

rubber band 4, k = 100 N.m-1 Fs = kx = 100 x 0.22 = 22 N

Calculate FII,

Fg = mg = 10 x 0.28 = 2.8 N,

90-23 = 67o, cos 67o = A/H = FII/Fg, FII = Fg x cos 67o, FII = 1.09 N

Applied force on cart = FII + Fs

rubber band 1, F = 3.476 + 1.09 = 4.57 N

rubber band 2, F = 6.754 + 1.09 = 7.84 N

rubber band 3, F = 17.89 +1.09 = 18.98 N

rubber band 4, F = 22+ 1.09 = 23.1 N

1. Calculate the velocity of the car when it reaches the horizontal plane.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
| **mathematically models energy transformation** | identifies mathematical relationships | calculates the total energy at the top of the ramp | calculates the total energy at the base of the ramp | calculates the velocity of the car |

Now GPE + EPE = KE

GPE = mg$∆h$, GPE = 0.28 x 10 x 0.45, GPE = 1.26 J

rubber band 1, Ek = Es1 + Eg = 0.382 + 1.26 = 1.642 J, v2 = 2x 1.642/0.28, v1 = 3.42 m.s-1

rubber band 2, Ek = Es1 + Eg = 0.743 + 1.26 = 2.00 J, v2 = 2x 2.00/0.28, v2 = 3.77 m.s-1

rubber band 3, Ek = Es1 + Eg = 1.967 + 1.26 = 3.227J, v2 = 2x 3.227/0.28, v3 = 4.80 m.s-1

rubber band 4, Ek = Es1 + Eg = 2.42 + 1.26 = 3.68 J, v2 = 2x 3.68/0.28, v4 = 5.127 m.s-1

# 3. Momentum

As the student assumes there is no friction, once the cart is on the horizontal plane it will travel at a constant velocity. On the horizontal plane the cart collides with a wooden block and the two objects are jammed together and move with a new constant velocity.

1. Explain how the momentum of the cart changes after it collides with the block by drawing an annotated diagram.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
| **demonstrates understanding** | identifies concepts and/or formulae | explains concepts and/or formulae | applies concepts and/or formula to context  | connects concepts and/or formula |

Students will draw a labelled diagram showing the cart and block before and after the collision. [1]

An annotated diagrams with explanations [1]

Diagram and explanation connects to the context [1]

Need to show that P1 = P2 and, P1 = m1v1, and P2 = (m1 + m2)v2

So the momentum of the system will remain the same, but the final velocity of the cart and block will be lower.[1]

1. Justify your observations in part a) by assigning numerical values and using relevant mathematical relationships to model the change in momentum.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
| **mathematically models change in momentum** | identifies mathematical relationships | inserts appropriate numerical values | compares final momentum with initial | evaluates the limitations of the model |

Students will calculate the initial momentum for the system using the given mass and the velocity they calculated in the previous question.

P1 = m1v1 [1]

As P1 = P2 , and P2 = (m1 + m2)v2 they then calculate the final velocity [1]

They compare the two velocities [1]

The limitations of the model is that as there is energy loss, the initial velocity is lower so the initial momentum will be lower and the momentum of the system will also be lower.

It will not necessarily be true that momentum is conserved as the cart and block will slow down so the momentum will be lost.

Mark awarded if students identify that both the final velocity and final momentum would be lower due to losses from friction.

Reference: A great articles on using Youngs modulus and understanding Hookes law

https://www.ndepscor.ndus.edu/fileadmin/ndus/ndepscor/SundayAcademy/2018-19SAHypothesisAndProductTestingRubberBandWriteUp.pdf